Mensionization Complementation

The Mathematics of Hermetic Alchemy

Section 9

Introduction to Waves, Harmonics, and Spinars

An α, β oppositional relationship is commonly found in all *wave* functions including *Harmonics*. *Harmonic Motion* can be best explained in terms of waves with their *unique* types of *motion*, *properties*, and *interactions*. In this topic we will touch on fundamental *wave* function's properties and explain the interactions in relation to the (α, β) *Hermetic Alchemy* oppositional system.

Generally speaking, there are many known types of waves in *Nature*; there are *ocean* waves, *sound* waves, *mechanical* waves, and on a higher *frequency* level, there are *radio waves, microwaves, infrared light waves, visible light waves, ultraviolet light waves, X-rays*, and *gamma rays*. However, any wave function can be *generically* described by the mathematical equation:

$$V = f \cdot \lambda$$

This equation lists a *trinity* of their more commonly known properties, *velocity*, *frequency*, and *wavelength*. The equation can be semantically stated as, "the *velocity* of the wave is equal to the *frequency times* the *wavelength*." *Frequency* is measured in units of *inverse* time $\left(\frac{1}{\sec}\right)$, where no units are shown in the numerator. It is presented in this form from the fact that the (1) in the numerator is considered *one cycle*. "A *cycle*" is a *one* (1) *unit process'* not a standard unit of measurement like *mass*, *time*, or *length*; the *cycle* depends upon *time* when determining its values (one unit *completion* of the process or *cycle* happens in every one unit of measured *time*).

$$frequency = \frac{cycles}{(unit\ \ time)} = \frac{1\ cycle}{1\sec} = \frac{1}{\sec}$$

Since the wave process is *periodic*, repeating itself over and over. It has a *frequency*; how frequent is this cycle happening? *Frequency* determines how often these oscillations occur in a measured *time* period. The *International Standard* unit of time is the "*second*." Therefore *frequency* is measured in units of *cycles* per *second* or at times cycles per unit time (*seconds*, *minutes*, *hours*, etc.).

This section will examine *two* classic oppositional types of waves, *Traveling Waves* and *Stationary Waves*, with a special emphasis on *Stationary Waves*. The section begins with a brief introduction to *Longitudinal Waves*, the base for a familiar *traveling* wave known as a *sound* wave, its *harmonics* are found in every aspect of our daily lives.

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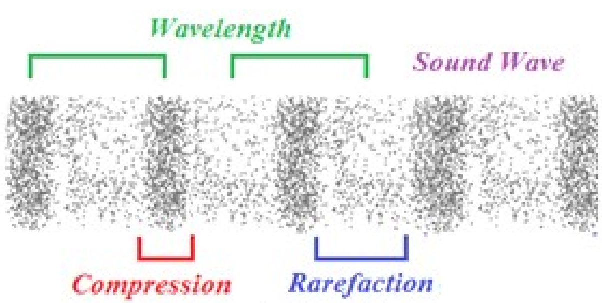
Longitudinal Waves

Sound Waves are moving *longitudinal* waves which result in the *medium* they travel through being parallel to the direction of the wave. Its properties are the α, β opposites *Compression* and *Rarefaction*. It has the same notations and properties as other waves which will be described as we progress.

Sound waves are composed of *moving air molecules*. Some molecules move close together (*Compression*) and other molecules move farther apart (*Rarefaction*). The following graphic shows the individual moving parts and their designations.



Longitudinal Wave

Sound waves properties are essentially equivalent mathematically to the other types of waves, however, the properties of the *medium* (*air*) they travel through plays the important role in their functioning. *Compressions* and *Rarefactions* are the equivalent of *Crests* and *Troughs* in *sine* waves. A sound wave's *energy* or *amplitude* is most often measured in terms of "*loudness*;" sound waves have *wavelength*, *amplitude*, *frequency*, and the properties of other waves as well.

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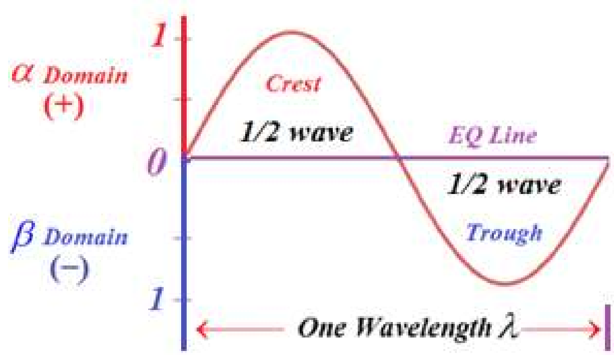
Moving or *Traveling* Waves

Traveling waves are better known as "*Moving Waves*." Moving waves include the ones you see and hear of most often; they are the free flowing, horizontal *Sinusoidal* type waves that travel at the speed of light in a vacuum.

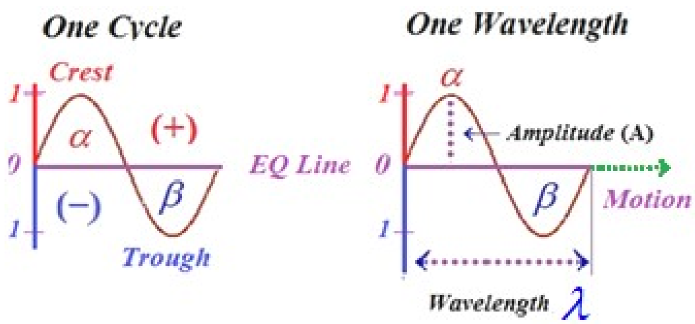
A *Sine Wave* is a geometric waveform that *oscillates* (moves *up* & *down*) in time and distance intervals, and is defined by the math function $y = A\ sin(kx - \omega t)$. In simpler words, it is an *s-shaped*, smooth curve that oscillates *above* and *below* a *zero* (0) *EQ Line*.

Sine Wave



It is divided into *two* (2) "Domains;" an (α) *positive* domain and a (β) *negative* domain. It shows the wave's structure of a *Crest* and a *Trough* that forms *one* (1) *wavelength*. The wavelength *lambda* (λ) is the *length* of *one* (1) complete *cycle*. Take note the *Crest* is ½-of the wavelength and the *Trough* is also ½-of the wavelength. A ½-wave is a *fundamental* beginning point in *harmonics*.

A sine wave's *wavelength* produces one "*cycle*" just as the 360° of a circle produces one *cycle*. *One* (1) unit cycle is shown as a (α) "*crest*" and (β) "*trough*." A wave mathematically has *amplitude*, *frequency*, and a *wavelength* as shown in the graphics below; the *amplitude* is a measure of the waves' *Energy* which is *proportional* to the *square* of its amplitude $E \propto A^2$.

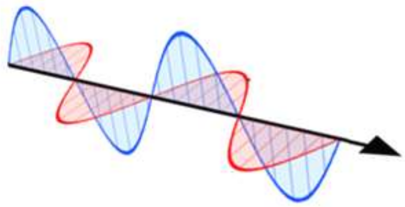


The *wavelength* is identified by the Greek symbol *Lambda* (λ), (however a capital (L) is used in determining other types of length in a fundamental wave system); the amplitude is a *wave-height* shown by a capital **(**A**)** or an upper-case Greek *alpha* as its symbol of identification.

Cyclic movement can be described by a simple analogy. When riding a *bicycle*, the pedals are on *opposite* sides of the bicycle for *balance*; they are also in 180° opposite positions to one another (one *up*, the other *down*). If you apply a *force* on a pedal with one foot (say the right foot); It goes from the *up* position to the *down* position; ½ of a circle (*cycle*). At the same instant you are applying a force with the *right* foot; the *left* pedal moves to the top ready for you to apply a force to it. *Kinetic Energy* is being *applied* by the right foot creating the necessary *Energy* for the bicycle to move; while at the same time creating the *Potential Energy* for the left foot by the left pedal rising to the top. The *right foot-left foot* cycle is a *periodic* cyclic movement that results in a *frequency*. The same *cyclic* process is repeated continuously until forced in some way to stop.

If it were not for *Damping*, the oscillations would never stop moving according to *Newton's* 1st *Law* of *Motion* (or at least until something or someone stops it). If no external force or energy is *available* to the system it would slowly lose its energy until the process would stop. This gradual loss of energy within a system is called a *Damping Effect*. Damping can be similarly compared to *aging* and *dying* in humans.

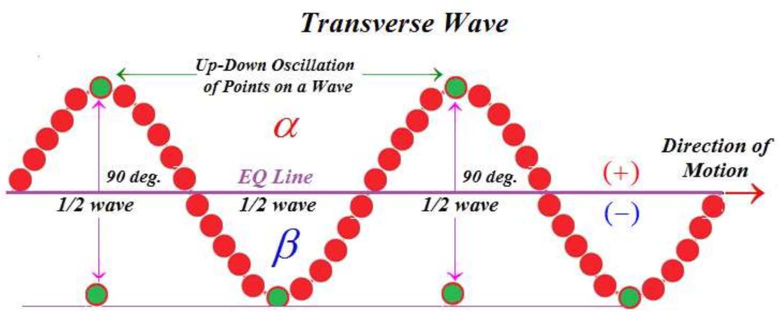
In this topic, we will use what are defined as *Transverse Waves*. A transverse wave is a *Moving* or *Traveling* wave which is basic to the functioning of Electromagnetic Waves (*EM waves*); the functional movement occurs at right angles (90° *perpendicular*) to the direction of the wave's *linear* motion; an *EM wave* also has a 90° lateral side *complementary* perpendicular movement carried along with it. The red wave is a *Magnetic* field and the blue wave is an *Electric* field.



Electromagnetic Wave

The following graphic will show some of the *transverse* wave's moving properties. In the graphic below, the *EQ line* is the equilibrium position between the upper *positive* and lower *negative* domains of the graphic. In numerical operations, the *upper* domain is referenced as *positive* or a (α) domain and the *lower* domain as *negative* or a (β) domain.

Each curved part of the wave (*crest* or *trough*) shown above and below the *EQ Line* is equivalent to *one*-*half* of a wavelength and is separated into individual position points shown as *red* and *green* colored circular points.

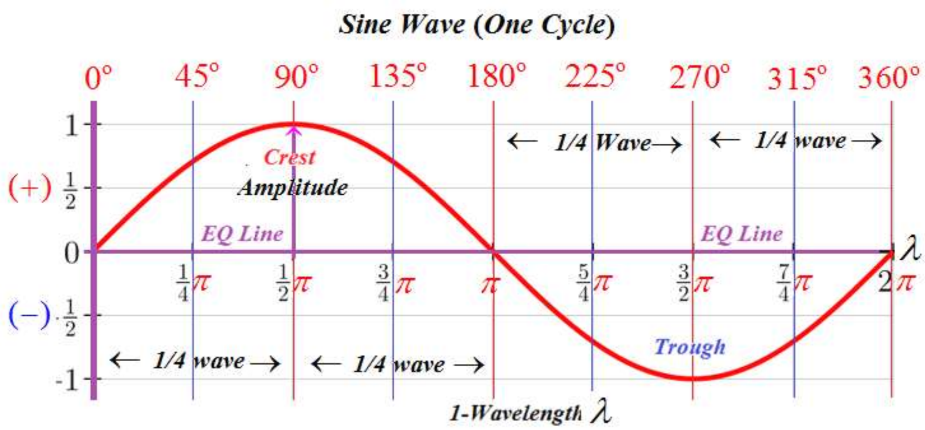


Each *green* colored point is shown at a 90° angle to the *EQ line*; this 90° *perpendicular* angle is the defining point of *moving* transverse waves. Each individual *red* or *green* point is *measured* by its 90° perpendicular displacement (*distance*) from the *EQ line*. These points do not move *forward* or *backward* out of their current *linear* position relative to the *EQ Line*, their movement is a perpendicular *up* and *down* oscillation *above* and *below* the *EQ line*. By the *up* and *down* movement of each *red* or *green* position point, the overall *simulation* of the wave gives an *appearance* of *moving* in a *right linear* direction. The particular wave does *not* move, the position points of the wave move *up* and *down*. The above wave is also referenced as a "*moving*" wave; it appears to move toward the right. *All* moving waves can be completely described by the *Sine* function $y = A\,sin(kx - \omega t)$ where A is the amplitude, $k = \frac{2\pi}{\lambda}$, λ = *wavelength* *and* $\omega = 2\pi f$.

Lateral 19

The Sine Wave

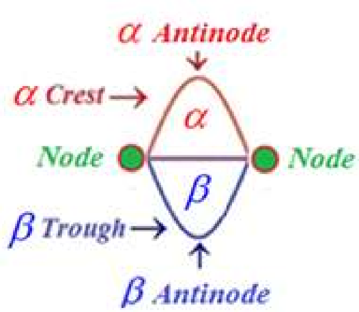
If a *circular* path is travelled we eventually return to an *original* starting point. At the original starting point, a *circular* cycle begins to repeat itself. The total distance traveled in *one* (1) cycle is the length of the *circumference* of a circle. Just as the *circumference* equals *one* cycle, one *wavelength* is also equivalent to *one* cycle. A *wavelength* includes a *crest* and a *trough*. Within the length of one *circumference* or one *wavelength*, four (4) *transformations* are occurring; each transformation occurs within ¼ of the *wavelength* and ¼ of the *total* distance traveled in a circle. The graphic below contains the *generic* information of a *sine* wave.



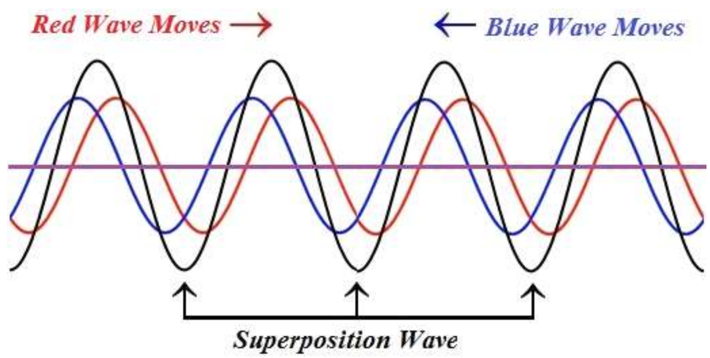
Lateral 20

The Origin of Standing Waves

Stationary waves are more commonly known as "*Standing Waves*;" they are in a class of their own; they interact between two *stationary* boundaries called "*Nodes*;" the waves cannot move outside of each boundary (*node*). All *interactions* must occur within the boundaries of the *starting* and *ending* nodes, shown below by the *red*-*circled* green nodes.

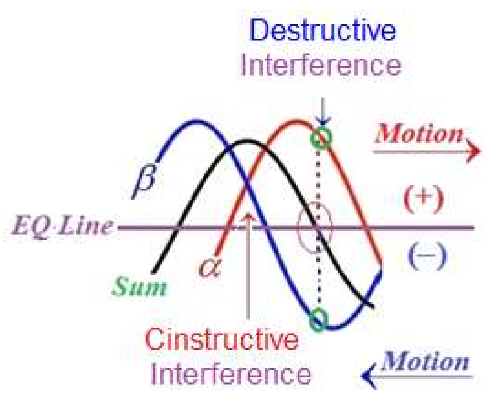


Standing waves have their origin in moving transverse waves which is a result of the combination of two waves moving in opposite directions, each having the same frequency, amplitude, and velocity. Since the standing wave can only propagate between the two stationary points or boundaries, when the red beginning or incident (α) wave hits the ending boundary it flips vertically and bounces back upon itself becoming a (β) "returning" wave traveling in the opposite (β) direction. The bouncing back results in two (2) waves moving in opposite directions which interfere with each other; the interference is called wave interference; in other words, when two beginning opposite moving waves are superimposed, their energies are either added together or canceled out creating a Superposition Wave.



The graphic above shows the result of the red and blue opposite moving waves being subjected to wave interference which generates the black Superposition Wave.

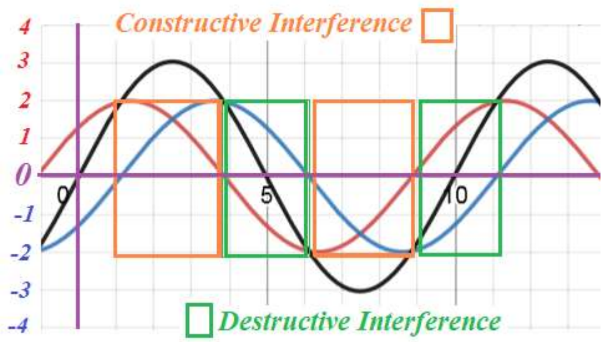
In the graphic below there are three (3) waves shown as a snapshot of their position at any one random point in time. The black superposition wave is the result of the red and blue waves' interference interactions. It is the result of a sum of any two corresponding position points of the red and blue waves relative to a specific length on the EQ line. Observe the two green circles in the graphic below; they occupy the same perpendicular linear position of length on the EQ line; the difference being, one is in a positive domain and the other is in a negative domain.



Addition of the two green point's + & – values or equal displacements from the EQ line will sum to zero (0). Therefore the black superposition point will change to where it exactly *crosses* the EQ line (zero point). While this is happening you will not visually see the red and blue waves, they will cancel out and leave only the black superposition wave visible. Their positive and negative energies have been canceled out and disappear.

The black superposition wave is the sum of any two points' perpendicular displacements relative to the EQ Line position. The sum of both positive and negative points is called "Destructive Interference."

"Constructive Interference" is formed when two position points are both in an upper positive (+) or lower negative (–) domain are summed. Just sum the two values and keep the same sign. In the graphic above, the thin red arrow shows the point where each of the red and blue waves occupy the same positive domain position when they cross paths, so their displacements are equal and would be summed producing a more elevated black or superposition location point.

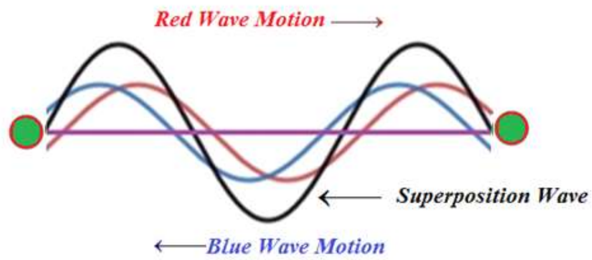


Constructive and Destructive Interference in Standing Waves

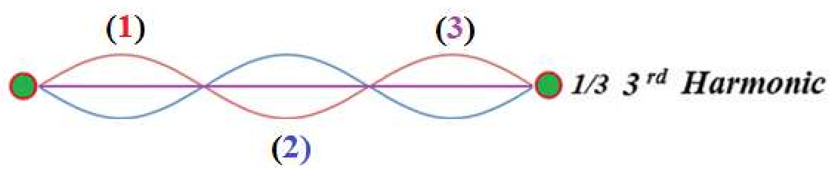
A little more in depth explanation is needed to show how the beginning interactions produce the dominant movements. It is how the superposition wave becomes the primary operational wave for all higher levels of future standing wave representations. NOTE: The black superposition waves will also be shown later on as red and blue waves to highlight their oppositional qualities.

Beginning with the standing waves' standard red and blue opposite moving interference waves; keep in mind the waves are moving against each other and as they bounce back forth between the green nodes, the waves cross paths (superimpose); they are constantly subjected to constructive and/or destructive interference in each linear unit position of the purple EQ Line.

Superposition Wave



If the above *three* (3) *antinode* small section of an extended transverse wave is placed between two immoveable objects (*green circles*); the standing wave shown above would become a 3rd *Harmonic* standing wave. By enclosing the section between two (2) barriers, it forces the *interior red* and *blue* waves to become *opposite* moving *waves*. The 3rd *harmonic* within the *Harmonic Series* is then formed. In the 3rd *harmonic* below the *red* and *blue waves* have now become superposition waves.

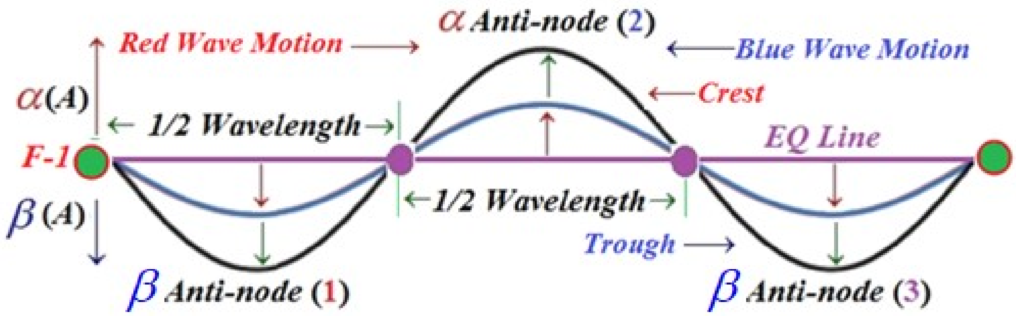


Note in the graphic above; the 3rd *harmonic* has *three* (3), ½ wave, Superposition *integral* Standing wave *antinode pairs* bound by the two *green* circles. The antinode numbering (**1**), (**2**), and (**3**) show each has an "*antinode pair*" associated with it. Antinodes (**1**) and (**2**) produce one *wavelength* that can be defined as containing 2, ½ *wavelengths* hence the *third wavelength* is also a ½ *wavelength*.

It should also be *noted* that each of the wave graphics in this section was created on the *Desmos* website by the author; just Google "*Desmos Wave Functions*" and you will have available a free educational *wave* program to create any type of wave function.

As the *interior red* and *blue* interference waves propagate between the green nodes, still shots (*Frames*) will be recorded showing the results of *constructive* and *destructive* interference within their lateral *horizontal* movement. Beginning with Frame-1, the graphic below identifies some wave properties and oppositional α, β movements. The beginning Superposition wave is shown as a black wave in these illustrations.

Frame (1)



A first observation of the graphic above only shows the blue (β) wave, the reason being, the *red* and *blue interference* waves are superimposed over each other; *blue*, being a dominant color, covers up the *red* wave in a computer monitor. Both the α & β *red* and *blue* interference waves share the same linear coordinate *positions*, *amplitudes*, *frequency*, and *velocity*.

Frame-2 shows the beginning separation of the *two* interference waves. The *red* and *blue* waves have the same consistent properties throughout their movements. Frame1 shows their maximum *amplitude* or *energy* level. The resulting *sum* of their values (*red* and *green* arrows) places the superposition antinode wave at its *maximum* peak value. Its peak *displacement* is a measure of its *energy*.

The standing wave is formed from three (3) ½-wavelengths and each *antinode pair* is given a number (**1**, **2**, and **3**). There is also a *sidebar* on the left side of each Frame (*shown right*) that contains two measuring arrows (*red* and *blue*) for the superposition waves' *amplitude* measurement in each Frame. The tips of the two arrows are lengths of the maximum value amplitude of the antinode *crests* and *troughs*. The α (A) & β (A) are an upper (α) and lower (β) positive or negative *amplitudes* divided into two (2) domains by the *EQ line*.

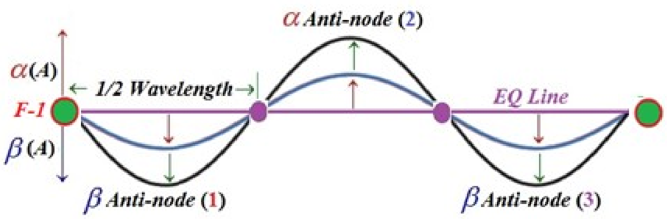
Frame Still Shots

Things to note in Frames **1** *&* **2**

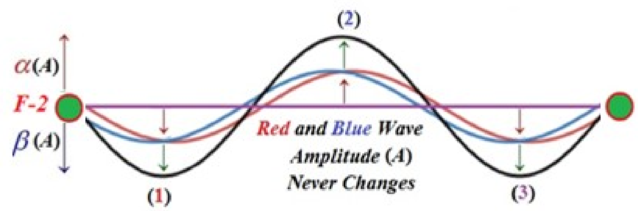
In the following still-shots:

- *Frame*-2 shows a beginning of movement and separation of the *red* and *blue* waves.
- Observe each antinode *crest* or *trough* in each of the frames to visualize their movement.
- Note that antinode (**1**) and antinode (**2**) oscillate in *inverse* directions to each other; a consistent alternation of (α) and (β) ½-wavelength *inverse* oscillations.

Frame (1)



Frame (2)

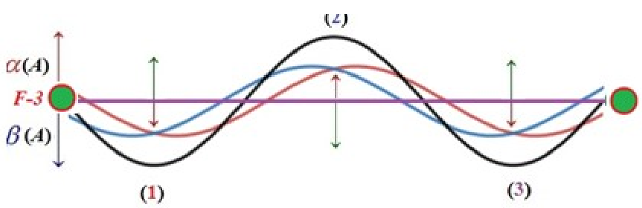


Things to note in Frames 3 & 4

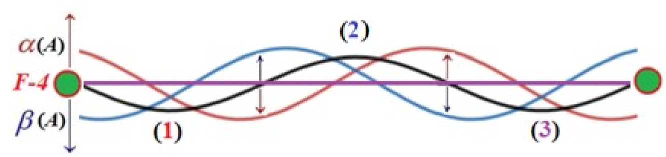
In the following still-shots:

- You will notice only a slight change in the *superposition* antinode *amplitudes* of Frames-1, 2, & 3.
- A marked *decrease* in the superposition wave's antinode amplitudes is shown in Frame-4.
- The arrows in Frame-3 highlight the *opposing* oscillations in each alternating ½-wavelength antinode α, β pair.
- Frame-4 is an excellent example of *destructive interference* between the *red* & *blue* waves.
- The rest of the frames are shown without comment to visualize the black superposition antinodes' vertical *up* and *down* movement.

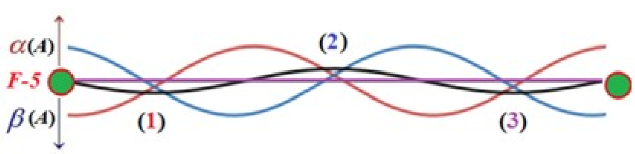
Frame (3)



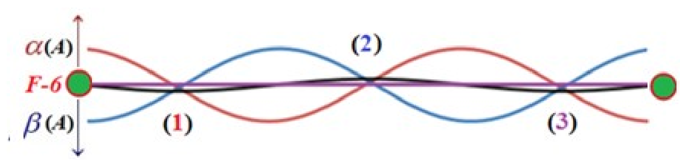
Frame (4)



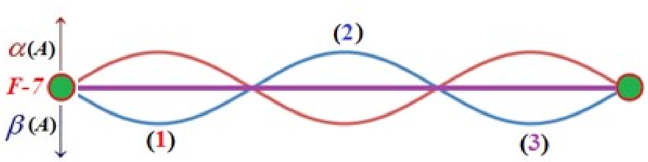
Frame (5)



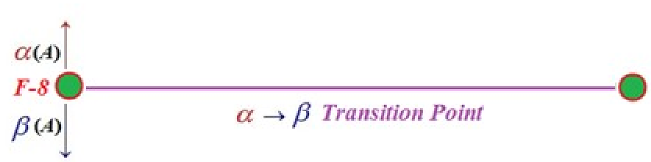
Frame (6)



Frame (7)

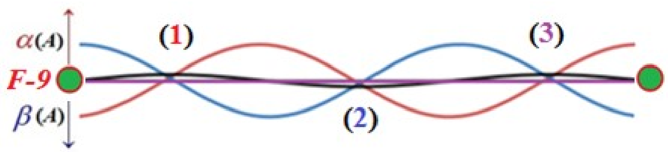


Frame (8)

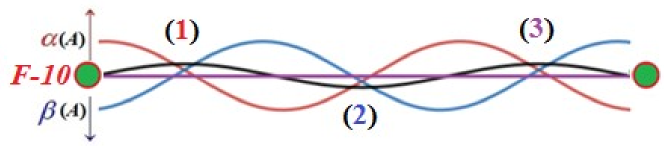


Frame-7 is the point of complete *destructive* interference; each of the individual points of the *red* and *blue* waves sum to *zero* which results in the blank wave shown in Frame-8. This position in the movement of the superposition antinodes is at a *Transition* point. When it crosses the *zero* line, the complete system will change into its *opposite*. Superposition antinodes (**1**) and (**3**) change from the *lower* (β) domain to the *upper* (α) domain; superposition antinode (**2**) changes from its (α) domain to its (β) domain.

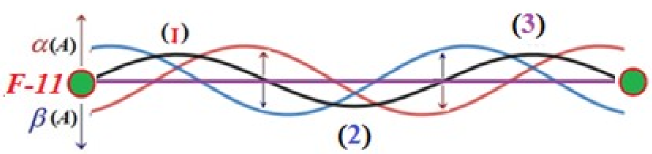
Frame (*9*)



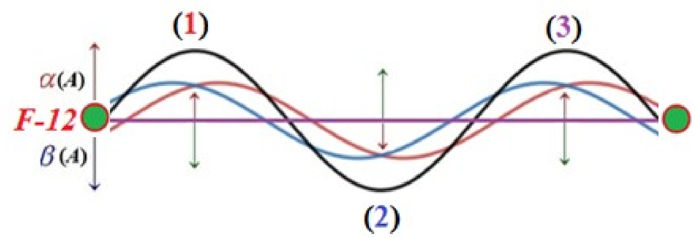
Frame (*10*)



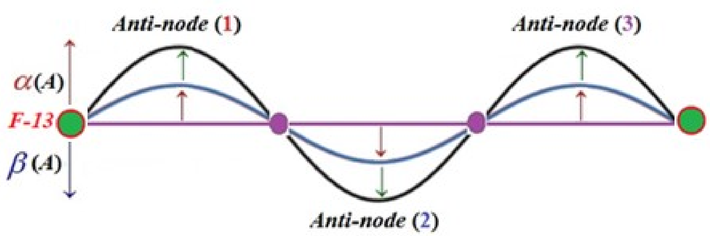
Frame (*11*)



Frame (*12*)



Frame-*13*



*Frame-*13 completes one-half (½) of the oscillation of each of the antinodes pairs (**1**), (**2**), and (**3**)'s movement. Just as in a *circle*, it has completed ½ of the *circle*. The *return trip* to its original *beginning* starting point is the *second* half and will complete *one* (1) oscillation. An important note to remember is the movements of the *red* and *blue horizontal* lateral waves' *constructive* and *destructive* interference resulted in the *up* and *down complementary* movement of the black *superposition's* antinodes.

Lateral 21

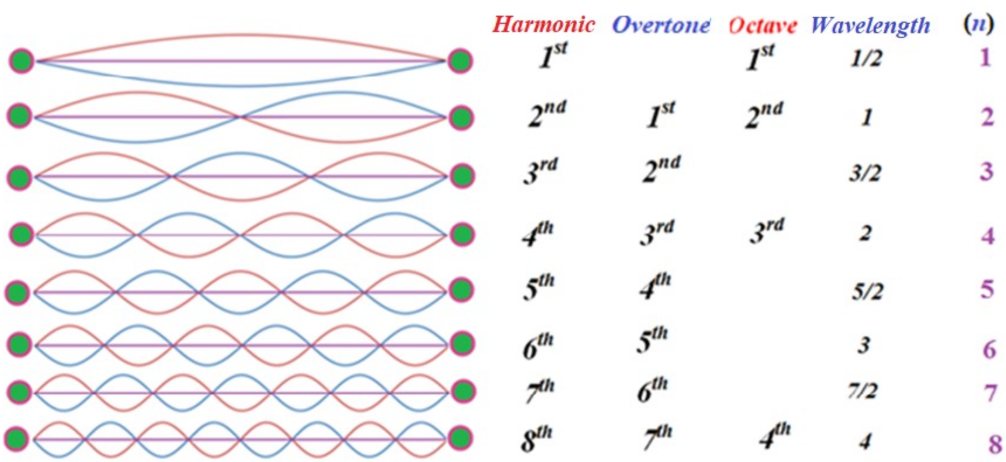
Fundamental Standing Wave Form

Standing waves *obey* the mathematics of a *harmonic* sequence known mathematically as the *Harmonic Series*. It is defined in mathematics as a sequence of numbers n_1, n_2, n_3, such that their *reciprocals* $\frac{1}{n_1}, \frac{1}{n_2}, \frac{1}{n_3}$, form an *arithmetic sequence*. Its' *series* is shown by the summation polynomial below.

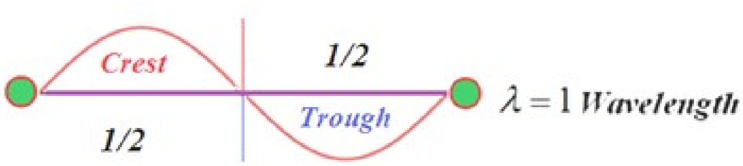
$$H_n = \sum_{n=1}^{\infty}\left(\frac{1}{n}\right) = \frac{1}{1} + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots + \frac{1}{n} + \cdot\cdot + \rightarrow \infty$$

The *Harmonic Series* consists of *waves* or *movements* whose *frequency* is an *integral* multiple of the *frequency* of the *Fundamental* 1st *Harmonic's* ½ wave. The following illustration shows the Series' graphic representations. The first set of *green* nodes below mark the boundaries of the *length* of the *Fundamental* 1st *Harmonic*. All other *harmonics* are *integral* multiples of the 1st *harmonic* and are to be referenced as *superposition* waves.

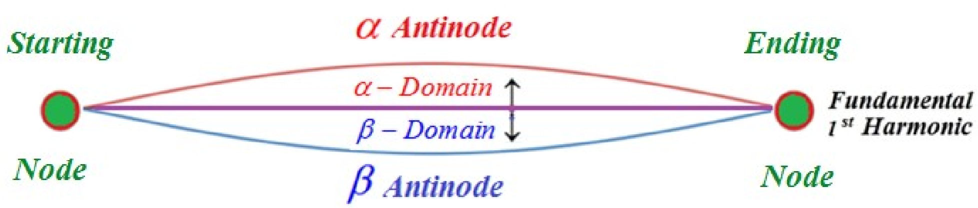
The Harmonic Series



An *important* relationship to keep in the back of your mind is that all *higher* harmonics *must* reside within the *length* of the *Fundamental* beginning 1st *Harmonic*; its *length* is the *length* of the Fundamental ½-*Wave* of *harmonics*. Normally *wavelengths* are measured in whole number cycles; where one complete *wavelength* consists of a "*crest*" and a "*trough*."



The *crest* is ½ of the *wavelength* and the *trough* is ½ of the *wavelength*. *Note* when displaying *harmonics*, a *trough* is included in the graphic of the *Fundamental* ½ wavelength shown below as an α, β inverse, the same is true with each of the other harmonics. Also note the *Fundamental* or 1st *Harmonic* ½-wave has a center dividing line. This dividing line is the purple *EQ line* shown in previous wave forms. It is the center dividing line between the α & β *harmonic* domains. The *up* and *down* double-headed arrow in the graphic below shows the primary direction of its *antinode pairs' up-down* oscillation. *Note* In the upcoming *Harmonics* below, the *red* and *blue* ½ waves are actually the previous black *superposition* waves that oscillate *up* and *down*. The colors are only to show the oppositions. Think of them as harmonic *superposition* waves.



In the graphic above, the upper *red* area is the (α) domain and the lower *blue* area is the (β) domain. In the α & β domain there are *two* oscillations occurring within the *Fundamental ½-wave*; although one of the oscillations is not observed from the graphic above. There is a *horizontal* wave movement occurring at the same instant as the *up-down antinode* movement; it is the previous *red* & *blue* interference waves' (α) and $(\beta)'s$ round trip moving *horizontally back* and *forth*; the incident wave bouncing *back* and *forth* between the nodes creating the opposite moving *interference* waves, in which the *superposition* waves are only shown in the graphic.

The *red* & *blue antinodes* occur in α, β *oppositional pairs*; the *red* & *blue* antinode pair shown in the graphic above is the maximum and minimum points of their *amplitudes*. They have previously been shown in the *still-shot* Frames.

Since the *Standing wave* is a *non*-traveling *wave*, the superposition waves' *up* and *down* movement gives the *appearance* of the *antinode* pair seem to be *standing still*, not moving out of its linear position.

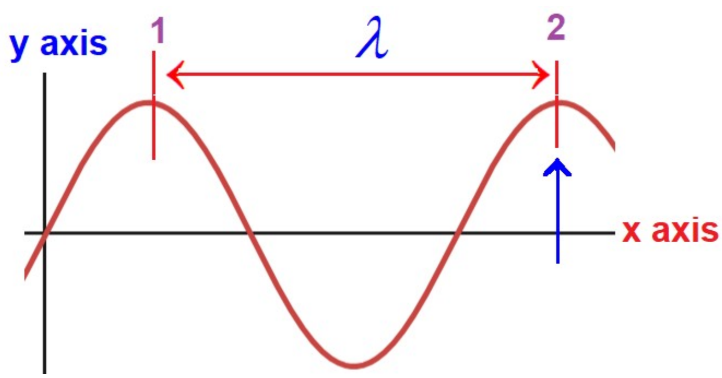
Because they appear to be *standing still* just moving up and down. personally I call them "*Spinars*." They just sit in one place and "oscillate up and down; they will be mentioned off and on, and discussed in the latter part of this section.

The Mathematics of Standing waves

As mentioned earlier in this section, Standing waves have their beginning in Moving waves and can be defined by the equation:

$$y = A\ sin\ (kx - \omega t)$$

Where A is the amplitude and $\omega = 2\pi f$ and $k = \frac{2\pi}{\lambda}$. Any traveling wave can be completely defined by the function above. The minus sign (-) in the $(kx - \omega t)$ of the general equation means the wave is traveling toward the right, a plus sign means it's traveling to the left or in the opposite direction. The graph of the sine wave is shown below.

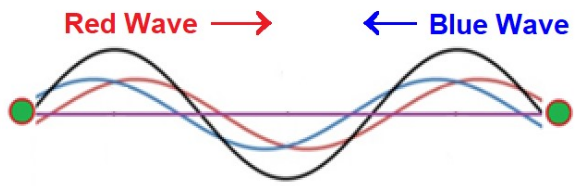


One wavelength, Lambda (λ) is shown as the distance between the two peaks 1 and 2. The wave is moving toward the right; if we have a stationary point, shown by the blue arrow; the number of peaks passing by the blue arrow in one (1) second is the frequency (f) of the traveling wave. The Period of the wave, written as $\mathrm{T} = \frac{1}{f}$, is one (1) divided by the frequency. It is the time it takes for one wavelength to pass the fixed blue arrow. The velocity of the wave is given by the equation $V = \lambda f$, or the wavelength times the frequency. Recall the equation for k is $k = \frac{2\pi}{\lambda}$. If we solve this equation for the wavelength (λ) we will get its equation $\lambda = \frac{2\pi}{k}$. Also if we use the equation for omega ($\omega = 2\pi f$) we can solve it for the frequency $f = \frac{\omega}{2\pi}$. We now have formulas for both the wavelength and frequency for the equation $V = \lambda f$. If we substitute the re-arranged equations in the formula we can solve for the Velocity. The equation would then become

$$V = \frac{2\pi}{k} \cdot \frac{\omega}{2\pi} = \frac{\cancel{2\pi}}{k} \cdot \frac{\omega}{\cancel{2\pi}} = \frac{\omega}{k}$$

where the 2π terms cancel out leaving us with the equation $V = \frac{\omega}{k}$. So when we have the general equation $y = A\ sin\,(kx - \omega t)$, the *velocity* is *omega* divided by *k.*

Let's return for a moment to the *red* and *blue interference* waves. We have *two* (2) *waves* that are



traveling in *opposite* directions; the *red* wave is traveling to the *right* and the *blue* wave is traveling to the *left.* If we name the equation for the *red* wave y_1 its equation would be $y_1 = A\,sin\,(kx - \omega t)$. If we change the *minus* sign of the *red* equation to a *plus* sign and assign it to the *blue* wave as y_2 it would change the direction of the *blue* wave motion to be traveling to the *left* and the *blue* wave's equation would become $y_2 = A\ sin(kx + \omega t)$. Recall that when two waves cross paths they superimpose and their values are summed. To get the "total effect" of the superimposing we sum the two equations where $y = y_1 + y_2$. From just the *geometry* of the summed equations our new equation becomes $y = 2A\,sin\,(kx)\cos(\omega t)$. This is the *equation* for a "*Standing Wave*." By changing the *frequency* of the standing wave we obtain the *Harmonics* in *the Harmonic Series*.

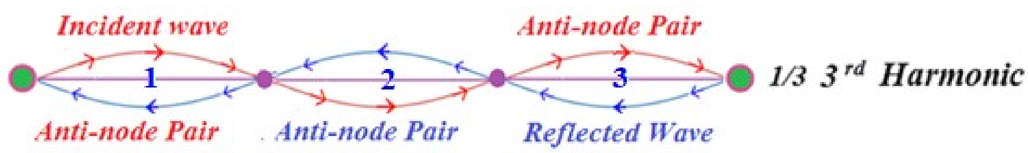
Quantization and Integralization

To *intuitively* understand the *Harmonic Series* we must *first* properly define its' polynomial equation and graphics. In physics there is a term called *Quantization*. The term was first introduced by *Niels Bohr* in his early 1900's explication on Earnest Rutherford's planetary model of the atom. Since that time it has been *refined* and *expanded* to the point it has become a *common occurrence* in almost all areas of physics.

I *personally* define *Quantization*, (in *harmonics)*, by the use of *two* terms, *Quantization* and *Integralization. Integralization* is *not* part of *standard* terminology, but is a kind of separation of function method. *Integralization*" reflects the process of imposing numerical order onto a continuous field. So while traditional physics only uses *quantization*, *Integralization* highlights how countable structure emerges within continuity. It *complements quantization*, does *not* replace it.

Quantization is the selection of allowed energy or frequency states. Integralization is the spatial structuring; how those quantized states manifest as geometric patterns with integer symmetry. They're like two sides of a harmonic coin: one defines which states are allowed, the other defines how those states take form.

To begin, I'll show how standing waves are quantized by use of the Fundamental 1st Harmonic's ½ Wave. Every n^{th} harmonic is a system of ½ waves. Let's begin with the 3rd harmonic that has 3, ½ wavelengths.



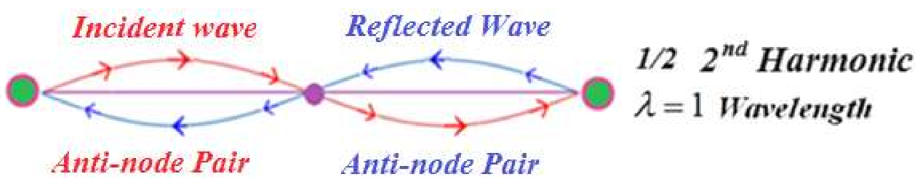
The two red circled green nodes represent the Fundamental 1st Harmonic's beginning ½ wavelength.
The 3rd harmonic is divided into three equal parts, where each of the three parts represents a ½-wave. Notice parts 1 & 2 define one wavelength as does parts 2 & 3 thus each part is ½ of a harmonic wavelength. This structure is the same in every n^{th} harmonic.

The above process is termed Integralization. Integralization is defined as "the act of combining or being combined into a cohesive whole." In this context, the word "whole" refers to the length of the Fundamental First Harmonic. The division aspect of the Harmonic sequence expressed through fractions, assigns a numerical value of one (1) to each numerator.

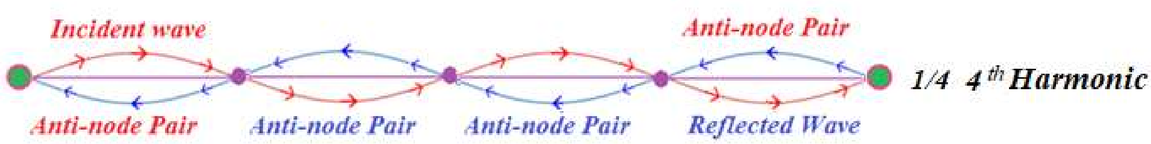
This "1" represents the act of dividing (integralizing) a standardized unit length—namely, the Fundamental ½ wave—into fractional integral parts. This base unit may be considered 5 inches, 10 meters, or any fixed length, so long as it defines the initial ½ wavelength used throughout the process.

These various Integralizations of a single-unit length yield different levels of the Harmonic Series. Beginning with the first fraction (1/1), the numerator (1) signifies a Fundamental ½-wave length that sets the standard, not to be exceeded when introducing additional integral parts or harmonics into the series.

The denominator of each fraction in the harmonic series has two (2) meanings. It first determines the number of *integral* parts the *Fundamental length* will be separated into and the *numerical value* itself is the *Harmonic number* (1st *Harmonic*, 2nd *harmonic, etc*). In the second term of the *Harmonic Series* polynomial above, the fraction ½-means *integralize* the standard *Fundamental length* into two (2) *equivalent integral* parts; we do not increase the *Fundamental* length. The 2nd harmonic establishes *one* (1) *interior* linear *wavelength* which can be *defined* as *two* (2) *integralized harmonic* ½ wavelengths.



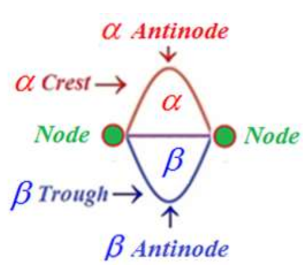
Similarly, the fraction (1/3) would *integralize* the unit length into *three* equal *integral parts* or sections and can be defined as an *Integralization* into *three* (3) equivalent *½ wavelengths*. The graphic for the 4th *Harmonic* is shown below.



Its *definition* includes being *integralized* into *two* (2) *linear wavelengths,* which can be thought of as having *four* (4), ½ wavelengths.

Quantization occurs because the *harmonics* in the *Harmonic Series* are *whole number* multiples of the number of ½ waves formed within each harmonic while *Integralization* defines the structural system.

These changing ½ *wavelengths* create α, β *antinode pairs*. A structure of each *antinode pair* is shown again below:

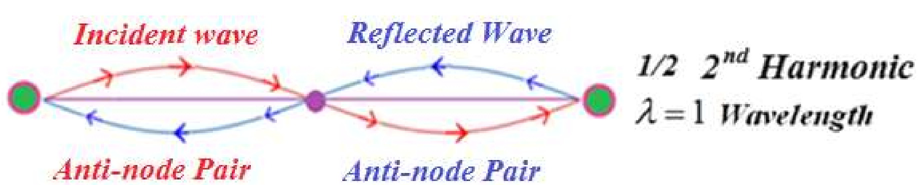


The *direction* of *propagation* in the antinode system above begins at the first node with an (α) *incident* or beginning wave; it rises to a *crest* (α *antinode*), and proceeds to the second node.

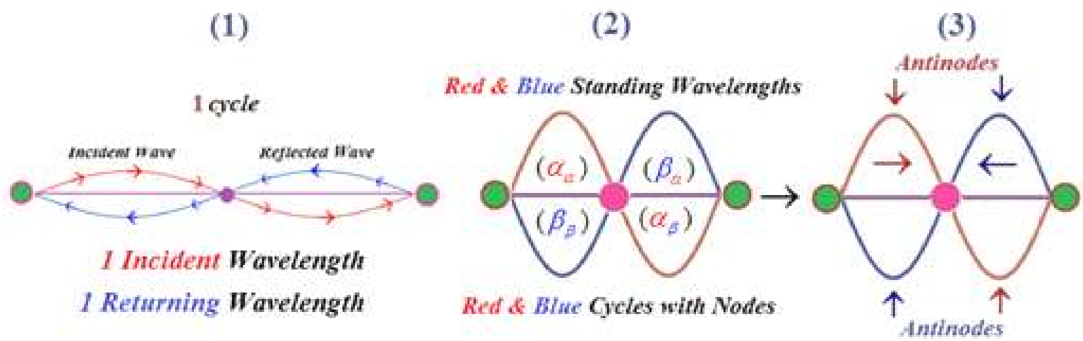
When the *red* wave hits the ending node it *flips vertically*, changes into its *opposite* (β *antinode*), proceeds in the opposite direction forming a *blue* identifying β *trough* then back to the first node. This route produces one *lateral* oscillation. If the oscillation took one second to complete; it would have a frequency of 1 oscillation (*cycle*) per second, or one Hertz (1 *Hz*). The oscillation above defines the waves' horizontal *lateral* movement, it is a *back* and *forth* cycle that produces superposition waves. It is not the primary oscillation of the standing wave. The primary movement in a standing wave is a *complementary up* and *down* superposition oscillation of the antinode pair formed from the wave interference encountered in the lateral cycle. We have already discussed how the antinodes oscillate by the movement of the *interior red* and *blue* waves' oppositional horizontal *lateral* movement forming superposition points in the individual frames of a moving wave.

The following paragraph will discuss the *decreasing* value of the *wavelength* in each individual n^{th} *harmonic*. The *wavelength* in the equation changes by the expression $f\uparrow \; n\uparrow \lambda\downarrow$ where "*n*" is the number of a*ntinode pairs*. As the *frequency increases* more antinode pairs are formed and the *wavelength decreases*. Each n^{th} harmonic is a multiple of the *Fundamental* 1st *Harmonic* so if the *frequency doubles*, the *wavelength* will decrease by one half (1/2); if the *frequency triples*, the *wavelength* will decrease by 1/3. In other words the *wavelength* will mathematically take care of itself by rearrangement of the formula $V = f\lambda$ to $\lambda = \frac{V}{f}$. The velocity in any medium will remain constant. The last video in the Waves' Video section will outline all the different formulas and show how they are used.

The 2nd Harmonic (1st Overtone)

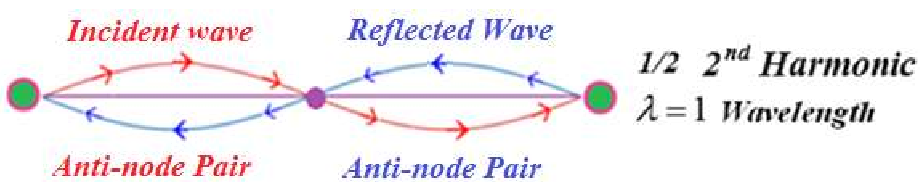


The first thing to note is the 2nd harmonic's *frequency* has *doubled* to two (2) *antinode pairs* and by *doubling* the *frequency*, the *wavelength* of each antinode pair is decreased by *one-half* of the original and has just *generically* divided the *Fundamental* ½-wave into *two* equivalent parts There are now *two* (2) ½-*wavelengths* which *sums* to one holistic *wavelength*.



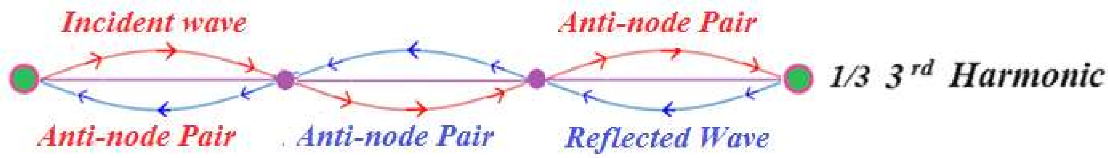
When bound into a standing wave, there are *two* (2) horizontal wavelengths formed, a α *red incident* wave and a β *blue returning* wave in which both waves *propagate* in opposite directions. Therefore, one *lateral* cycle now contains two (2) antinode pairs and a length of two (2), ½-*wavelengths* to propagate *per lateral horizontal cycle*.

Note below we have added a center *purple* node*. Purple* nodes occur whenever the *red* and *blue* waves cross the zero *EQ Line* at the exact same position of length (*complete destructive* wave interference at that point).



The 2nd harmonic is also termed the 1st *overtone*. The easiest way to define an overtone is through the word *resonance*. When two or more waves *resonate* it means they have some *properties* in *common*. Using the strings of a piano as an analogy; if a certain low note is played on a piano, it will cause the other strings in the piano which have *frequencies* in common with the original note to oscillate in sync with the note being played. These *resonances'* add together producing a *robust*, sweet, *full sounding note*. These different *resonant frequencies* are called *overtones*. Excluding the Fundamental *1st Harmonic*, *each subsequent n*th harmonic in the *Harmonic Series is also an overtone* of the *fundamental harmonic*. Overtones do not have overtones; their *frequencies* are formed of pure *sinusoidal* waves. The 3rd *Harmonic* in which we will soon discuss is also the 2nd *overtone.* If the fundamental note is played, it can cause both the 2nd *harmonic* (1st *overtone*) and 3rd *harmonic* (2nd *overtone*) to oscillate and produce sound that is added to and blended with the *Fundamental* harmonic. Overtones add *quality*, *timbre*, and *energy* to *harmonics*.

The 3rd Harmonic (2nd Overtone)



The 3rd *Harmonic* is considered a *separation* point between the 2nd *octave* and 3rd *octave* in the Harmonic Series. The 3rd *octave* begins with the formation of the 4th *Harmonic*. Even though the *Harmonic Series* is an *integralized* sequence, it also aligns with the *Hermetic Alchemists'* (2^n) *exponential* expansion in the creation of "*octaves*." The *Hermetic Alchemy* (2^n) expansion is shown again below:

$$2^n = (2^0 = 1) + (2^1 = 2) + (2^2 = 4) + (2^3 = 8) + \cdots + \to \infty$$

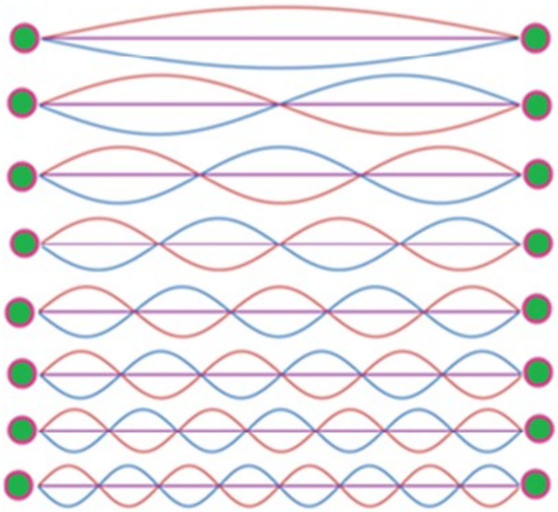
Both the *integralized harmonic* sequence and the (2^n) sequence are infinite sequences. By rule of *equivalency*, the (2^n), expansion can be *blended* with the arithmetic sequence to define its *octave* occurrences. Each time a *harmonic doubles* in its *antinode pairs* relative to the *Fundamental*, it creates an *octave* in *harmonics*. In the (2^n) expansion each term *doubles* the previous term's numerical value, thus creating a sequence of *octaves* usable in *harmonics*. The polynomial below is an *equivalence blend* of the (2^n) expansion with the *Harmonic Series.* It highlights the occurrence of the *octaves* within the *harmonic* series.

$$H_n = \sum_{n=1}^{\infty}\left(\frac{1}{n}\right) = \frac{1}{2^0} + \frac{1}{2^1} + \frac{1}{3} + \frac{1}{2^2} + \frac{1}{5} + \frac{1}{6} + \frac{1}{7} + \frac{1}{2^3} + \cdots + \frac{1}{n}$$

Lateral 22

Harmonic Integralizations (Spinars)

The Harmonic Series



Harmonic	Overtone	Octave	Wavelength	(n)
1st		1st	1/2	1
2nd	1st	2nd	1	2
3rd	2nd		3/2	3
4th	3rd	3rd	2	4
5th	4th		5/2	5
6th	5th		3	6
7th	6th		7/2	7
8th	7th	4th	4	8

Each n^{th} *harmonic* above is an n^{th} *dimensional spinar.* To *properly* explain *spinars* we must first use *successive derivatives* of an n^{th} harmonic standing wave. Successive derivatives will reveal how many lower dimensional *spinars* are contained within that particular n^{th} dimensional level. For example, in the upcoming section on *Differential Dimensions*, it is explained how *Differential Dimensions* are related to the *exponents* of dimensional variables. A *brief* explanation is shown below of a beginning 5^{th} *Harmonic* from the *Harmonic Series* above which is also termed the 5^{th} *Differential dimension* or 5^{th} *exponential* of the *Alchemical* $f(m_5) = (\alpha + \beta)^5$ dimension.

5^{th} *Harmonic Successive Derivative Levels*

0^{th}	1^{st}	2^{nd}	3^{rd}	4^{th}		5^{th}

$$1(\alpha+\beta)^5 \rightarrow 5(\alpha+\beta)^4 \rightarrow 20(\alpha+\beta)^3 \rightarrow 60(\alpha+\beta)^2 \rightarrow 120(\alpha+\beta)^1 \rightleftharpoons (5!)(\alpha+\beta)^0$$

The 0^{th} *derivative* above is the original *beginning* point of the *spinar*, in the above case it is an identifying beginning point $1(\alpha+\beta)^5$, no *derivatives* are taken in the 0^{th} *derivative.* The 1^{st} *derivative* shows there are (5) 4-*dimensional spinars* $5(\alpha+\beta)^4$ and the other *successive derivatives* give twenty (20) 3-*dimensional spinars* $20(\alpha+\beta)^3$; sixty (60) *2-dimienional spinars* $60(\alpha+\beta)^2$; and 120, 1-*dimensional spinars* $120(\alpha+\beta)^1$ all contained within the 5^{th} *Harmonic.*

Because the primary movement of a *spinar* is the *up* and *down* oscillation of its superposition *antinode pairs*, it does *not* have the appearance of traveling. It remains static in its relative position and constantly oscillates *up* and *down*.

Each *antinode pair* is an $(\alpha+\beta)^1$ binomial. The 5th *Harmonic* would be equivalent to $(\alpha+\beta)^5$ It has *five* (5), *½ integral multiple wavelengths* where each of these (5), *½ integral multiple wavelengths* is an α,β *antinode pair*. We can then treat each *n*th *harmonic set* of *antinode pairs* mathematically as *exponential dimensions*.

In *Section 10* we will put it all together forming α,β *Differential Dimensions*.

Differential Dimensions in Section 10.