



Mensionization Complementation

The Mathematics of Hermetic Alchemy

Section 4

The Preliminary Mathematics

7. Understanding the Calculus math for this Section.

Since this explication was written with the *general population* in mind, the *mathematics* may be like a foreign language to some readers; however I will present an easy way to understand this section's use of two (2) principles of *calculus* math. To those of you who are not familiar with, nor have had a calculus course; for this short tutorial all you need to have is knowledge of a few rules of *algebra*. If you've had *pre-cal* or a *calculus* course, you're all set to begin.



There is a term that needs to be properly defined before we begin. It is "*Dimension*." The word dimension can have many different meanings. In this manuscript, it will be used to reference *two* (2) concepts, *dimensions* of *space* and *exponents* of *variables*.

Dimensions of Space are mostly self-evident; however, variables' *exponents* are the heart of *algebraic* manipulations. An *exponent* is often termed a *dimension*. If you increase the *integer* numerical *value* of a variable's *exponent* you increase the variables' *energy* and *range of influence* or *dimension*. For example, in the *exponential* (2^1), the integer value two (2) is the base and the exponent is one (1); it is an \mathbb{R}^1 1-dimensional; it references the *duality* (2) of a line which has an oppositional *beginning* and *end*, it has *two* (2) *inherent* properties, a *Principium* (α) and *Principius* (β) its range is ($2^1 = 2$). If you increase the *exponent* to a value of *two* 2, (2^2), it becomes a \mathbb{R}^2 2- dimensional; (it has four (4) *inherent* properties; you multiply the base which is two (2), *times* itself (2 x 2). If the *exponent* of two (2) is increased to the third power or exponent (2^3), it becomes an \mathbb{R}^3 3-dimensional; it has eight (8) *inherent*

properties or $(2 \times 2 \times 2)$. Each time you increase the *exponent* one integer unit value you *double* the *range* of its *influence*. It is the basic structural math of *Hermetic Alchemy*.

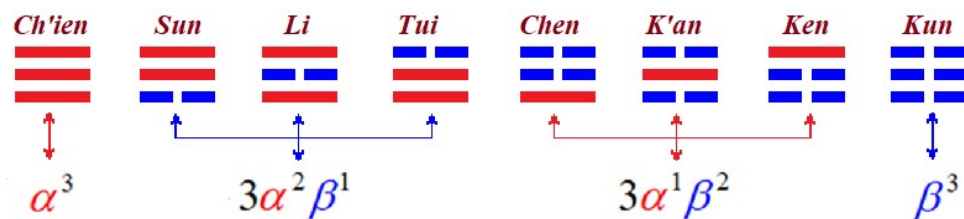
In \mathbb{R}^n *orthogonal* spatial dimensions, 1-dimension of space 2^1 is a line with an oppositional *beginning* and *end* that has *one* (1) of three (3) different orientations in space, a *front* to *back* binary orientation, a *side* to *side* binary orientation, or an *up* and *down* binary orientation; a 2-dimension (2^2), is a square *plane* with *two* lines that are complemented *orthogonally* (90°) to each other which defines an “*Area*,” and a 3- dimension (2^3), is *three* (3) lines that are complemented *orthogonally* to each other; holistically it is also known as *volume* or *space*.

In *Hermetic Alchemy* a 1-dimensional *opposition* is a binomial set ($2^1 = 2$) of Base 2 α, β oppositions (the *Volatile* (α) and *Fixed* (β)); 2-dimensions ($2^2 = 4$), contain the sets of *Four Elements* (*Fire, Earth, Air, and Water*). 3-dimensions ($2^3 = 8$), is considered one (1) *Trinity* that has eight (8) properties as in the eight (8) *trigrams* of the *I Ching*.

In the *Eastern I Ching*, a 1-dimension is the set of the Base 2 α, β oppositions *Yin* broken line () and *Yang* solid line (). A 2-dimension is also analogous to the *Four Elements* of the Alchemists. They are known as the *4-Hsiang* in the *I Ching*. Both are shown in the graphics below. In the *I Ching* they are commonly referred to as *Bigrams* which consists of a ratio of two *positions* (one above the other) that can hold either a solid or broken line.



A 3- dimension is also the set of eight (8) *Trigrams* ($2^3 = 8$) shown in the graphic below with their exponential α, β *Pascal* multiplication nature.



Now that we have the above data, we can explain how *calculus* math transforms the α, β oppositional system. In the above discussion *exponents* were emphasized. We will use *calculus* math to take advantage of *two* (2) of its main properties, *differentiation* (*derivatives*) and *Integration* (*Integrals*). By taking the *derivative* of a function we can *lower* an *exponent* by one (1) *integer* unit or *1-dimension* and with *integration* we can *increase* a function's *exponent* by one (1) *integer* unit or *1-dimension*; thus giving us a *mathematical* way to manipulate an α, β function's *exponent* or *dimension*. For example, using the 3-dimensional oppositional $f(m_3) = (\alpha + \beta)^3$ function; we can take a "first" *derivative* of the $(\alpha + \beta)^3$ using the *Chain Rule* and *lower* its exponent by one (1) *integer* unit using the formula $n(\alpha + \beta)^{n-1}$. The result will become $f(m_2) = 3(\alpha + \beta)^2$. The *coefficient* of the $(\alpha + \beta)^2$ is three (3) which means the result will consist of three (3) different *permutations* or *arrangements* of the 2-dimensional $(\alpha + \beta)^2$. The original 3-dimensional opposition $(\alpha + \beta)^3$ was changed from a 3-dimensional function to *three* (3) *arrangements* of 2-dimensional functions $3(\alpha + \beta)^2$. The derivative process will mathematically calculate the permutations within its formula $n(\alpha + \beta)^{n-1}$. To show the derivative process in the real world; the "first" *derivative* of the 3-dimensional opposition $(\alpha + \beta)^3$ results in the *Alchemical three* (3) *Tria Prima*, 2- dimensional (*Sulphur*, *Salt* and *Mercury*) functions. There is much more involved in the derivative process than just lowering an exponent's value though, however, when I now mention the term "*derivative*" you will know I am basically lowering some function's *exponential* or dimensional value by one *integer* unit; you won't be completely confused in the process.

The next principle of *calculus* is *Integration*. *Differentiation* and *Integration* are inverses of each other just as *addition* and *subtraction* are inverses; they "*undo*" each other. The process of *Integration* *increases* a function's *exponent* by one *integer* unit. It changes the function to a higher dimension (increasing its *energy* and *range of influence*). Its symbol is the integral (\int)

sign and its formula is $\left(\frac{\alpha^{n+1}}{(n+1)} \right)$. The numerator of the formula is simply adding one (1)

integer unit to the currently existing exponent (n) and the denominator or divisor ($n+1$) is removing the previous exponents' *permutations*. An example will show this operation. In the

derivative of $f'(\alpha^3) = 3(\alpha^2)$, we change the 3-dimensional (α^3) to three (3) 2-dimensionals $3\alpha^2$ by the formula ($n\alpha^{n-1}$). The numerical coefficient three (3) is the number of permutations of the (α^2). If we integrate the $3(\alpha^2)$ derivative, we obtain the original value (α^3). It will have “undone” the derivative’s permutations and returned to its previous state.

$$\int 3(\alpha^2) d\alpha = 3 \int (\alpha^2) d\alpha = \frac{3 \cdot \alpha^{2+1}}{2+1} = \frac{\cancel{3} \cdot \alpha^3}{\cancel{3}} = \alpha^3$$

For your understanding of the calculus mathematics used in this manuscript, the derivative will basically be used to mathematically decrease a function’s exponent or dimension by one (1) integer unit and the integration function will be used to mathematically increase a functions’ exponent or dimension by one (1) integer unit. The two functions we will be using are Hermetic Alchemy’s (α) Generative Power (G) variable and the (β) Productive Capacity (P) variable.

To progress to a more advanced level of the preliminary mathematics, there are some additional materials we will need to have in hand so as to establish an orderly sequence in the math. Once we have the prerequisites in hand, we can then formulate a “General Polynomial Summation Equation” termed the “Duality of Form Polynomial.” The following summation equation is the projected final result from this section’s next three topics on the preliminary mathematics.

$$E(\alpha + \beta)^n = \sum_{k=0}^n \left\{ \frac{d^k}{d\alpha^n} (\alpha)^n \cdot \int_k \left(\frac{\beta^0}{0!} \right) d\beta \right\}$$

The Duality of Form Summation Polynomial

The Duality of Form Summation Polynomial is a name I applied to the equation above and is referenced as such throughout this manuscript. In reality the equation is just a Calculus version of algebra’s Binomial Equation. We will begin with the original binomial equation and proceed from there.

Binomial Equation

$$(x + y)^n = \frac{x^n y^0}{0!} + \frac{nx^{n-1} y^1}{1!} + \frac{n(n-1)x^{n-2} y^2}{2!} + \frac{n(n-1)(n-2)x^{n-3} y^3}{3!} + \dots + \frac{\cancel{n!} x^0 y^n}{\cancel{n!}}$$

In the *polynomial* above the x and y are the *algebraic complementary* variables. In the next polynomial we will replace the x and y with the *complementary* opposites α and β .

$$(\alpha + \beta)^n = \frac{\alpha^n \beta^0}{0!} + \frac{n\alpha^{n-1}\beta^1}{1!} + \frac{n(n-1)\alpha^{n-2}\beta^2}{2!} + \frac{n(n-1)(n-2)\alpha^{n-3}\beta^3}{3!} + \dots + \frac{\cancel{n!}\alpha^0\beta^n}{\cancel{n!}}$$

To complete the transition, there needs to be a *separation* of the *variables* to represent this particular type of equation.

$$(\alpha + \beta)^n = \alpha^n \cdot \left(\frac{\beta^0}{0!}\right) + n\alpha^{n-1} \cdot \left(\frac{\beta^1}{1!}\right) + n(n-1)\alpha^{n-2} \cdot \left(\frac{\beta^2}{2!}\right) + n(n-1)(n-2)\alpha^{n-3} \cdot \left(\frac{\beta^3}{3!}\right) + \dots + \cancel{n!}\alpha^0 \cdot \left(\frac{\beta^n}{\cancel{n!}}\right)$$

We now have the *binomial equation* in a form where we can begin identifying the *mathematical* interactions taking place within the polynomial. I'm sure you have noticed that I put the *factorial* progression beneath the β^n term, placing it in this particular position will be explained later in the text at a more appropriate time, however *first* lets observe the α^n as it progresses from *term* to *term*. Terms 2, 3, & 4 will explain the progression of the α^n variable.

$$\begin{array}{ccccccc} \alpha^n \cdot \left(\frac{\beta^0}{0!}\right) & + & n\alpha^{n-1} \cdot \left(\frac{\beta^1}{1!}\right) & + & n(n-1)\alpha^{n-2} \cdot \left(\frac{\beta^2}{2!}\right) & + & n(n-1)(n-2)\alpha^{n-3} \cdot \left(\frac{\beta^3}{3!}\right) & + & \dots & + & \cancel{n!}\alpha^0 \cdot \left(\frac{\beta^n}{\cancel{n!}}\right) \\ \text{Term 1} & & \text{Term 2} & & \text{Term 3} & & \text{Term 4} & & & & \end{array}$$

The first *term* $\alpha^n \cdot \left(\frac{\beta^0}{0!}\right)$ is the beginning or constant term of the polynomial. **Note* term 2*

contains the *coefficient* of α as $n\alpha^{n-1}$ which you will recognize as the *formula* for the *first derivative* of α^n ; *term 3*, contains $n(n-1)\alpha^{n-2}$ which is the *second* successive *derivative* of α^n and *term 4* α coefficient is $n(n-1)(n-2)\alpha^{n-3}$ which is the *third* successive *derivative* of α^n . The last term of the polynomial is simply the n^{th} term $\cancel{n!}\alpha^0 \cdot \left(\frac{\beta^n}{\cancel{n!}}\right)$ where the $n!$

coefficient cancels out in the expansion. In fact each *factorial* that is not equal to *one* (1)

cancels out. To show how the operation works, the following is an expansion of $(\alpha + \beta)^3$.

$$\begin{aligned}
 (\alpha + \beta)^3 &= \frac{\alpha^3 \beta^0}{0!} + \frac{3\alpha^2 \beta^1}{1!} + \frac{3(2)\alpha^1 \beta^2}{2!} + \frac{3(2)(1)\alpha^0 \beta^3}{3!} \\
 (\alpha + \beta)^3 &= \frac{\alpha^3 \beta^0}{\cancel{0!}} + \frac{3\alpha^2 \beta^1}{\cancel{1!}} + \frac{3 \cdot \cancel{2} \alpha^1 \beta^2}{\cancel{2!}} + \frac{\cancel{3} \alpha^0 \beta^3}{\cancel{3!}} \\
 (\alpha + \beta)^3 &= \alpha^3 \beta^0 + 3\alpha^2 \beta^1 + 3\alpha^1 \beta^2 + \alpha^0 \beta^3
 \end{aligned}$$

In the middle equation the $0!$ and $1!$ are equal to *one* (1) which has no effect on the equation, so we can cancel out these two *factorials*; however the higher *factorials* also cancel out.

The *holistic* result is just *Successive Derivatives* of α^n until α reaches its *zero exponent*.

Successive derivatives can be shown mathematically as a *summation* (Σ), where sigma's summation properties *force* the derivative's polynomial to become *successive derivatives*. It can be shown by the *calculus* term:

$$\sum_{k=0}^n \frac{d^k}{d\alpha^n} (\alpha)^n$$

We now have the first half of the *Calculus Binomial* or *Duality of Form* equation; however, there are *two* remaining interactions which must be resolved, – the interactions of the β *productive capacity* along with its *factorials*. The *two* remaining interactions will require properties that are available within the *integration* process.

There is a very important mathematical series/polynomial that needs to be presented at this point. It is the *companion* series/polynomial of the universal *Euler* constant (e). The $f(e^x)$ series-polynomial is a *Natural* or a *Nature* order of things, i.e., Nature's order of evolving; it will be defined in the next topic as a *successive Integration* series (IS), where we will explain how *Nature's* mathematics help temper *entropy* to create order and structure in our universe.

Since the original *Binomial Equation* is in polynomial form; the resulting polynomial will be what is termed a "*Power Form* Polynomial" (*exponents*). Polynomials actually make math much easier and more understandable. The polynomials will also show a more structural form of α, β Base 2 oppositions.

To begin the *prerequisite* information on the β *productive Capacity* and the *factorials* we will *first* need the assistance of *a very special form* of the $f(e^x)$ series in which we will *derive* at this point.

8. The *Successive Integration Series*- (*Expansion of the virtual* (β^0) *Productive Capacity*).

In this topic, the process used to derive the *successive Integration* series will be shown, along with how it is used mathematically. With mathematical *integration*, the different polynomial terms within each individual integration, along with each *exponent* increase, we will show the evolution of the (β^0) *Productive Capacity* as it increasingly takes on new, higher dimensional α, β oppositional properties and identifying data presences or elements. Before beginning the integration process of the *Successive Integration* Series (IS), some basic guidelines that will be followed in the *integration* process are listed below. *Integration* begins with the *Fundamental Unit Power Equation*:

$$\frac{x^0}{(0!)} = 1 ; x \neq 0, [(0!) \text{ is Optional}]$$

The *Unit Power Equation* is an extension form equation of the *Zero-Exponent Rule*, which states that any *non-zero* variable to the *zero-exponent* is a value that is equivalent to an *integer* value of *one* (1).

The Zero (0) Exponent Rule

The *zero (0) exponent* rule can be *proven* mathematically by use of *exponential division* and the *Transitive Law of Equality*.

Suppose that α , along with x and y , are *Numbers* or *Variables*. Where α is a common *base* and x & y are the exponents. When a *number* is raised to the power of 0, it presents a *special* situation that involves the rules of exponential *division*. According to these rules, the *quotient* of *two powers* with the *same base* is equal to the common *base* raised to the *exponent* that is

equal to the *difference* between the *exponents* x and y , $\left(\frac{\alpha^x}{\alpha^y} = \alpha^{x-y} \right)$. The *special* situation

arises when the *exponents* have the *same* numerical value. So, if $x = y$, where $x=1$, & $y=1$,

from the equation $\frac{\alpha^x}{\alpha^y} = \alpha^{x-y}$; then $\frac{\alpha^1}{\alpha^1} = \alpha^{1-1} = \alpha^0$. Also, by *regular* arithmetic division

$\frac{\alpha^1}{\alpha^1} = 1$. Therefore, by the *Transitive* property of *Equality*, $\alpha^0 = 1$. The result of the *Transitive*

property of *Equality* shows that *numbers* or *variables* raised to the power of *zero* (0) is equal to 1.

The *Unit Power Equation* shown above, in this case, is also thought of as being in a "0-dimensional *incorporeal* state" or a "*Prima Materia*" state. We will first substitute the virtual *Productive Capacity prima materia* (β^0) of the *Hermetic Alchemists* in the beginning equation for (x^0), where (β^0) is in a 0-dimension *incorporeal* state which can be easily expanded dimensionally and oppositionally. Another reason for starting with this form of the unit power

equation $\frac{\beta^0}{0!} = 1$; $\beta \neq 0$ is *first*, (β^0) is in its virtual state, it can easily adjust itself to conform

to whatever individual oppositional *quality* the *Alpha* (α) *Generative Power* happens to be; *second*, in the math, since the original *Binomial Equation's* terms were divided by *factorials* we can have *integration* begin with either *variables* and *integers* (*default*) or *variables* and *factorials*. To achieve our desired end result in the *integration* process we will need a *progression* of increasing *factorials* in the denominator of each individual integration; therefore, we will need to be using *variables* and *factorials*, which mean we will include the optional (0! *zero factorial*). The $(0!) = 1$ will help solve the factorial problem at the very beginning.

To prevent unnecessary misunderstandings we need to resolve *two* (2) situations before beginning the integration process; (1); we will need to algebraically *divide* each term in the resulting polynomial by a sequence of increasing *integer* multiple *factorials*; (2); in addition, we need to confirm $(0!) = 1$, to satisfy ourselves it is mathematically achievable. In the following equations, we will show $(0!) = 1$ and from its equality help solve the division by *factorials* situation. There are many ways to prove $(0!) = 1$ however the following equality proof is one of the simplest. We begin with a *factorial* equality.

$$(n!) = \frac{(n+1)!}{(n+1)} = \frac{(n!) \cdot \cancel{(n+1)}}{\cancel{(n+1)}} = (n!).$$

We then set $n = 0$ in the equality $(n!) = \frac{(n+1)!}{(n+1)}$.

$$\text{When } n = 0 \rightarrow (0!) = \frac{(0+1)!}{(0+1)} = \frac{1!}{1} = \frac{1}{1} = 1 \therefore (0!) = 1$$

Now that we've shown $(0!) = 1$ is mathematically achievable from a *factorial* equality; instead of just always hearing a professor's explanation of "(by convention $0! = 1$)"; we will have to review only one other often used property of factorials along the way to change division by *integers* to division by *factorials* throughout the *integration* process.

Beginning with the *Hermetic Alchemical Productive Capacity* form of the unit power equation, $\frac{\beta^0}{0!}$; from this term, a *successive Integration* series will be obtained. Other guidelines are as

follows; the first *term* of the polynomial will be $\frac{\beta^0}{0!}$. We will then *integrate* this *first* term. The *integration* of the *first* term will become the *second* term in the polynomial. *Integrate* the *second* term and it will become the *third* term; *integrate* the *third* term and it will become the *fourth* term, *ad infinitum*. The *result* will be a polynomial consisting of *Successive Integration*, where any *term* in the polynomial is an *integration* of the *previous integration* term, excluding the previous *term* of the beginning *term*. In addition, if any $f(x)$ can be shown by a *series*, and the *series* converges in an *interval* containing (x) , then inside that *interval*, we can *differentiate* or *integrate* "term-by-term." The properties of *term-by-term* integration using *successive integration* will, in this case, produce *two* different forms of polynomials, a *Power* form polynomial and an *Integration* form polynomial. The *integration* process begins showing the *two* forms of the *results* from the *integration* of the $\frac{\beta^0}{0!}$ term which is a beginning *constant* term in each polynomial.

Power Form

$$IS_1 = \frac{\beta^0}{(0)!}$$

Integral-Form

$$IS_1 = \frac{\beta^0}{(0)!}$$

Taking the *indefinite integral* of the first term with the *General Power Rule* $\int x dx = \frac{x^{n+1}}{n+1} + C$;

term 2 of the polynomials above then becomes the *integration* of the first term, $\frac{\beta^0}{0!}$.

$$\int \frac{\beta^0}{0!} d\beta = \frac{1}{(0!)} \int \beta^0 d\beta = \frac{1}{(0!)} \cdot \frac{\beta^{(0+1)}}{(0+1)} = \frac{\beta^1}{(0!) \cdot 1} = \frac{\beta^1}{(1!)} + C$$

NOTE: Since $0! = 1$, the integral $\int \frac{\beta^0}{0!} d\beta$ can also be equated to $\int \frac{\beta^0}{1} d\beta$ or just

$\int \beta^0 d\beta = \frac{\beta^{(0+1)}}{(0+1)} = \frac{\beta^1}{1}$ and the end result will be $\frac{\beta^1}{1}$, not $\frac{\beta^1}{(1!)}$ the one we need. However, in

$(4!) \cdot 5 = (5!)$
$(3!) \cdot 4 = (4!)$
$(2!) \cdot 3 = (3!)$
$(1!) \cdot 2 = (2!)$
$(0!) \cdot 1 = (1!)$

the above integration, carrying the first *factorial* (0!) in its true *factorial* form throughout the *integration* process, we can take advantage of an often used property of *factorials* as shown in the colored part of the table on the left $(0!) \cdot 1 = (1!)$, thus *converting* the denominator, integer one (1), to a (1!) without using *alternative* means.

This property is the equation $(n!) \times (n+1) = (n+1)!$. In the first *integration* above, the *conversion* took place in the $\frac{\beta^1}{(0!) \cdot 1}$ term and the process will be used throughout the

successive integration process. For example, the math below is a successive *triple Integration* showing the properties in the *table* above left.

$$\int \int \int \left(\frac{\beta^0}{(0!)} \right) d\beta = \left(\frac{1}{0!} \int \beta^0 d\beta = \frac{\beta^{0+1}}{(0!) \cdot 1} = \frac{\beta^1}{(1!)} \right) \rightarrow \left(\frac{1}{(1!)} \int \beta^1 d\beta = \frac{\beta^{1+1}}{(1!) \cdot 2} = \frac{\beta^2}{(2!)} \right) \rightarrow \left(\frac{1}{(2!)} \int \beta^2 d\beta = \frac{\beta^{2+1}}{(2!) \cdot 3} = \frac{\beta^3}{(3!)} \right)$$

First integration

Second integration

Third integration

The first *two* terms of (IS) will then become:

$$IS_2 = \frac{\beta^0}{(0!)} + \frac{\beta^1}{(1!)} \quad \text{or} \quad IS_2 = \frac{\beta^0}{(0!)} + \int \frac{\beta^0}{(0!)} d\beta$$

Power Form

Integral-Form

Before continuing, observe *two* notes; from this point on, in future integration, for *simplicity* and to avoid confusing *clutter*, we have *discontinued* the use of the *constant* of *integration* (+C) and will show that it is equal to *zero* (0) when the integration process is complete. The *third* term of (IS) will be the *integration* of the *second* term, so *integration* of the *second* term $\frac{\beta^1}{(1!)}$ we obtain:

$$\int \int \frac{(\beta)^0}{(0!)} d\beta = \int \frac{\beta^1}{(1!)} d\beta = \frac{1}{(1!)} \int \beta^1 d\beta = \frac{1}{(1!)} \cdot \frac{\beta^{(1+1)}}{(1+1)} = \frac{\beta^2}{(1!) \cdot 2} = \frac{\beta^2}{(2!)}$$

Integration of the second term, $\frac{\beta^1}{(1!)}$ results in the third term $\frac{\beta^2}{(2!)}$.

Listed below are now the first *three* terms of (IS):

$$IS_3 = \frac{\beta^0}{(0!)} + \frac{\beta^1}{(1!)} + \frac{\beta^2}{(2!)} \quad \text{and} \quad IS_3 = \frac{\beta^0}{0!} + \int \frac{\beta^0}{0!} d\beta + \int \int \frac{\beta^0}{0!} d\beta$$

Power-Form *Integral-Form*

The *third* term, above, in the *Integral-Form* of the polynomial $\left(\int \int \frac{\beta^0}{0!} d\beta \right)$, is shown as a double integration. The double integration sign is “the *integration* of the *integration* of the *first* term $\frac{\beta^0}{0!}$. The double integration sign in this term is known as “*Successive Integration*,” which can

be written in the simplified form of “*multiple*” integration symbols $\int \int \frac{\beta^0}{0!} d\beta$.

I’m sure you have observed the pattern that is developing in both the *power*-form and the *integral*-form of the two equivalent polynomials. We will take it one step further and show the fourth term of the polynomial confirms the observation. The *fourth* term is the integration of the *third* term. The *third* term is $\frac{\beta^2}{(2!)}$, so the integration of this term will give the *fourth* term.

$$\int \int \int \frac{\beta^0}{0!} d\beta = \int \frac{\beta^2}{(2!)} d\beta = \frac{1}{(2!)} \int \beta^2 d\beta = \frac{1}{(2!)} \cdot \frac{\beta^{(2+1)}}{(2+1)} = \frac{\beta^3}{(2!) \cdot 3} = \frac{\beta^3}{(3!)}$$

The *fourth* term is $\frac{\beta^3}{3!}$, so the *power*-form and *integral*-form become:

$$IS_4 = \frac{\beta^0}{(0!)} + \frac{\beta^1}{(1!)} + \frac{\beta^2}{(2!)} + \frac{\beta^3}{(3!)} \quad \text{and} \quad IS_4 = \frac{\beta^0}{0!} + \int \frac{\beta^0}{0!} d\beta + \iint \frac{\beta^0}{0!} d\beta + \iiint \frac{\beta^0}{0!} d\beta$$

Power-Form *Integral-Form*

We won't do any further integration, however, the pattern does continue *infinitely*. Therefore, the result in the first case is a *power-form* polynomial on the order of:

$$(IS_{n+1}) = \frac{\beta^0}{(0!)} + \frac{\beta^1}{(1!)} + \frac{\beta^2}{(2!)} + \frac{\beta^3}{(3!)} + \frac{\beta^4}{(4!)} + \dots + \frac{\beta^n}{(n!)}$$

The above polynomial can be rewritten as the *power form* of the *MacLaurin* Series for $f(e^\beta)$

$$f(e^\beta) = \frac{\beta^0}{(0!)} + \frac{\beta^1}{(1!)} + \frac{\beta^2}{(2!)} + \frac{\beta^3}{(3!)} + \frac{\beta^4}{(4!)} + \frac{\beta^5}{(5!)} + \dots + \frac{\beta^n}{(n!)} + \dots \rightarrow \infty$$

This *power form* polynomial can also be written as a *summation* polynomial.

$$f(e^\beta) = \sum_{k=0}^n \left(\frac{\beta^k}{k!} \right) = \frac{\beta^0}{0!} + \frac{\beta^1}{1!} + \frac{\beta^2}{2!} + \dots + \frac{\beta^n}{n!}$$

To resolve the *Constant of Integration*, we can use the *calculus* rule that in a series or polynomial, the *integral* of a *sum* is the *sum* of its *integrals*, which can be shown by the following example summation equations.

$$f(x) = \int \sum_{n=0}^{\infty} c_n(x^n) dx \quad f(x) = \sum_{n=0}^{\infty} \int c_n(x^n) dx \quad f(x) = C + \sum_{n=0}^{\infty} \frac{1}{n!} \left(\frac{x^{n+1}}{n+1} \right)$$

Integral of a Sum = *Sum of its Integrals* = *Resulting Sum*

We obtain a *Constant of Integration outside* the series, which in this case we would have:

$$f(IS) = C + \left\{ \frac{\beta^0}{0!} + \frac{\beta^1}{1!} + \frac{\beta^2}{2!} + \frac{\beta^3}{3!} + \frac{\beta^4}{4!} + \dots + \frac{\beta^n}{n!} + \dots \rightarrow \infty \right\}$$

$$f(IS) = C + f(e^\beta)$$

Taking the *1st-derivative* of $f(IS)$ we obtain a *Constant of Integration* equal to *zero* (0).

$$f'(IS) = f'(C) + f' \left[f(e^\beta) \right] = 0 + f(e^\beta) \quad \therefore f(IS) = f'(IS) = f(e^\beta)$$

We can now define the expansion of the *Hermetic Alchemical Productive Capacity* (β) principle as a function $f(e^\beta)$ by use of *Successive Integration*, where each *successive integration* term is placed beneath its corresponding equivalent *power* term in the polynomial equations we just derived.

$$f(e^\beta) = \frac{\beta^0}{(0!)} + \frac{\beta^1}{(1!)} + \frac{\beta^2}{(2!)} + \frac{\beta^3}{(3!)} + \frac{\beta^4}{(4!)} + \dots + \frac{\beta^n}{n!}$$

$$f(e^\beta) = \int_0^{\beta^0} \frac{\beta^0}{0!} d\beta + \int \frac{\beta^0}{0!} d\beta + \iint \frac{\beta^0}{0!} d\beta + \iiint \frac{\beta^0}{0!} d\beta + \int \int \int \frac{\beta^0}{0!} d\beta + \dots + \int_n \frac{\beta^0}{0!} d\beta$$

One of the more *powerful* and *useful* polynomials in *Mensional Mathematics* may be obtained by *Successive Integration* of a *Unit Power Equation*. However, this *Integration Series* is a very special case. It is a *successive integration* process resulting in a function type $f(e^x)$. The above resulting $f(e^\beta)$ polynomial is known as a “*natural* polynomial or a *Nature* polynomial.” By that, I mean it follows *Nature’s* laws of natural evolution. On a *fundamental* level, it is a way of saying; “it’s how *Nature’s* mathematics *temper entropy* to bring order to things.”

9. The *Successive Differentiation Series*

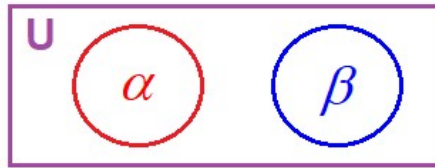
As was mentioned earlier in the text, *integration* and *differentiation* are *inverse* operations in the same way *addition* and *subtraction* are *inverse* operations. Since the integral’s operation $\int(e^\beta) = f(e^\beta)$ is equivalent to the derivative’s $f^1(e^\beta) = f(e^\beta)$, both produce the same *output*. We can show that *successive integration* of the *Alpha* (α) *Hermetic Generative Power* principle could also be in a form of:

$$f(e^\alpha) = \frac{\alpha^0}{(0!)} + \frac{\alpha^1}{(1!)} + \frac{\alpha^2}{(2!)} + \frac{\alpha^3}{(3!)} + \frac{\alpha^4}{(4!)} + \dots + \frac{\alpha^n}{n!}$$

$$f(e^\beta) = \frac{\beta^0}{(0!)} + \frac{\beta^1}{(1!)} + \frac{\beta^2}{(2!)} + \frac{\beta^3}{(3!)} + \frac{\beta^4}{(4!)} + \dots + \frac{\beta^n}{n!}$$

Thus defining the *Hermetic alpha* and *beta* or semantically termed *Generative Power* and *Productive Capacity* as equivalent forms of one another. Placing the (α) and (β) one above the other as shown above, mathematically, confirms the *Alchemical* statement in the *Hermetic*

Kybalion on opposites “*like* and *unlike* are mathematically the same.” If both are mathematically equivalent functions, then to be considered an *opposition*, there must be something else involved for the two to be classified as *oppositions*. In fact, there is, and its name is *Complementation*. (α) and (β) are mathematically true *complementary opposites*. For example, *complementation* essentially means to make something *whole* or *perfect*. If we have a universal *set* containing only the α *Generative Power* and the β *Productive Capacity* then this set contains *two* (2) *complements*.



A *complement* in a set is essentially its “*opposite*” within the defined universal set. The β complement contains all the elements in the universal set that are not in the original α set, it is everything else that makes it *whole* or *perfect*.

Since we have shown the existence of a *Successive Integration* series/polynomial in the first part of this section; there also exists a simple form of a *Successive Differentiation* series-polynomial.

As we advance further into mathematics the use of *successive differentiation* and *integration* becomes more prominent in defining physical *hierarchies*. *Speed*, *Velocity*, and *Acceleration* are some of the more basic ones.

To put the *successive derivative* series in its most simple polynomial form, we must alter the polynomial with the help of the *Binomial Theorem*. Notice from the derivative power formula ($n\alpha^{n-1}$); when we perform a derivative and decrease the *exponent* by α^{n-1} , we are changing the “*dimension*” and lowering the *energy* and *range* of the output. For example, if we were to take the *first derivative* of (α^3), its output would be ($3\alpha^2$); we have changed the *dimension* from a 3- dimensional exponent of three (3) to a 2- dimensional exponent of two (2). The derivative of the variable determines the number of *permutations* of the (α^2) result. This decrease in *dimension* is actually telling us there are *three* (3) different *permutations* of the 2-

dimensional (α^2) in complementary oppositions. In [Section 6](#), It will be shown *how* this decrease in dimension affects the *Hermetic Alchemy Four Elements*, however, for now, I will present a *summation proof* by induction of the following equality, i.e., *permutations* are equivalent to a successive derivative's *coefficients*. Permutations can be found within the *Binomial Theorem*.

$$\sum_{k=0}^n \left\{ {}_n P_k \text{ or } \left(\frac{n!}{(n-k)!} \right) \cdot \alpha^{(n-k)} \right\} = \sum_{k=0}^n \left\{ \frac{d^k}{d\alpha^n} \alpha^n \right\}$$

$$\sum_{k=0}^n {}_n P_k (\alpha^{n-k}) = \sum_{k=0}^n \frac{d^k}{d\alpha^n} \alpha^n$$

We will begin by letting ($n=3$) dimensions and show each summation index term (k) for $k=0$ to 3 :

$$\sum_{k=0}^3 \left\{ \left(\frac{n!}{(n-k)!} \right) \cdot \alpha^{(n-k)} \right\} = \sum_{k=0}^3 \left\{ \frac{d^k}{d\alpha^n} \alpha^n \right\}$$

When ($k=0$), no derivatives are performed.

$$[k=0] \quad \sum_{k=0}^3 \left\{ \frac{3!}{(3-0)!} \cdot \alpha^{3-0} \right\} = \frac{3!}{3!} \alpha^3 = 1\alpha^3 = \sum_{k=0}^3 \left\{ \frac{d^0}{d\alpha^3} \alpha^3 \right\} = 1\alpha^3$$

$$[k=1] \quad \sum_{k=0}^3 \left\{ \frac{3!}{(3-1)!} \cdot \alpha^{3-1} \right\} = \frac{3 \cdot 2!}{2!} \alpha^2 = 3\alpha^2 = \sum_{k=0}^3 \left\{ \frac{d^1}{d\alpha^3} \alpha^3 \right\} = 3\alpha^2$$

$$[k=2] \quad \sum_{k=0}^3 \left\{ \frac{3!}{(3-2)!} \cdot \alpha^{3-2} \right\} = \frac{3!}{1!} \alpha^1 = 6\alpha^1 = \sum_{k=0}^3 \left\{ \frac{d^2}{d\alpha^3} \alpha^3 \right\} = 6\alpha^1$$

$$[k=3] \quad \sum_{k=0}^3 \left\{ \frac{3!}{(3-3)!} \cdot \alpha^{3-3} \right\} = \frac{3!}{0!} \alpha^0 = 6\alpha^0 = \sum_{k=0}^3 \left\{ \frac{d^3}{d\alpha^3} \alpha^3 \right\} = 6\alpha^0 \text{ or } (3!)\alpha^0$$

Showing *permutations* are equivalent to the *coefficient* of the derivatives allows us to formulate a simple form of a *successive derivative* polynomial, that does *not* use *derivatives*.

$$\sum_{k=0}^n \frac{d^k}{d\alpha^n} \alpha^n = \sum_{k=0}^n \left\{ \frac{n!}{(n-k)!} \alpha^{n-k} \right\} = \frac{n!}{(n-0)!} \alpha^{(n-0)} + \frac{n!}{(n-1)!} \alpha^{n-1} + \frac{n!}{(n-2)!} \alpha^{n-2} + \dots + \frac{n!}{(n-k)!} \alpha^{n-k}$$

The *coefficient* of each polynomial term is the *factorial formula* for the number of *Permutations* of the *changed* variable. Observe the following math; we are going to take three *successive*

derivatives of (α^3) by the polynomial above, $n=3$ & $k=0 \rightarrow 3$. Remember when $(k=0)$ no derivative is performed.

$$\begin{aligned} (k=0) \quad & \frac{n!}{(n-k)} \alpha^{(n-k)} = \frac{3!}{(3-0)!} \alpha^{(3-0)} = \frac{\cancel{3!}}{\cancel{3!}} \alpha^3 = \alpha^3 \\ (k=1) \quad & \frac{n!}{(n-k)} \alpha^{(n-k)} = \frac{3!}{(3-1)!} \alpha^{(3-1)} = \frac{3 \cdot \cancel{2!}}{\cancel{2!}} \alpha^2 = 3\alpha^2 \\ (k=2) \quad & \frac{n!}{(n-k)} \alpha^{(n-k)} = \frac{3!}{(3-2)!} \alpha^{(3-2)} = \frac{3!}{\cancel{1!}} \alpha^1 = 6\alpha^1 \\ (k=3) \quad & \frac{n!}{(n-k)} \alpha^{(n-k)} = \frac{3!}{(3-3)!} \alpha^{(3-3)} = \frac{3!}{\emptyset!} \alpha^0 = 6\alpha^0 \end{aligned}$$

We obtain the same values as in the proof above. **Special Note:** The *Successive Integration Series* and the *Successive Differentiation Series* are the base operational *functions* of the “*Duality of Form* polynomial.” We have now solved the second half of the *Duality of Form* or *calculus* version of the *Binomial Equation*. It is shown in its expanded form below using both variables.

$$\begin{aligned} E(\alpha + \beta)^n = & \alpha^n \cdot \int_0 \left(\frac{\beta^0}{0!} \right) + n\alpha^{n-1} \cdot \int_1 \left(\frac{\beta^0}{0!} \right) + n(n-1)\alpha^{n-2} \cdot \int_2 \left(\frac{\beta^0}{0!} \right) + \\ & n(n-1)(n-2)\alpha^{n-3} \cdot \int_3 \left(\frac{\beta^0}{0!} \right) + \dots + \cancel{n!} \alpha^0 \cdot \left(\frac{\beta^n}{\cancel{n!}} \right) \end{aligned}$$

9. The *Duality of Form Polynomial*.

The *Binomial Theorem* is a very important process in the mathematics of series and polynomials and mathematics in general. Its evolution in math began several millennia in the past as far back as 4 BCE. Its mathematics involves the expansions of the $(\alpha + \beta)^n$ binomial and α, β oppositional functions such as the *Lattice Datum* system, *Hermetic Alchemy*, and the *I Ching*.

In the beginning through the middle parts of *Section 2*, we discussed the inner operations of a binomial expansion and from those inner operations of the binomial expansion an *inference* suggested the use of successive differentiation multiplied by successive integration of the

$(\alpha + \beta)^n$. This inference was the reasoning behind the formation of the “*Duality of Form* summation polynomial.” The final output produced a summation polynomial equation which is shown below in its current form. It is a *calculus* version of algebra’s *Binomial* Equation. It uses both *successive differentiation* and *successive integration*.

$$E(\alpha + \beta)^n = \sum_{k=0}^n \left\{ \frac{d^k}{d\alpha^n} (\alpha)^n \cdot \int_k \left(\frac{\beta^0}{0!} \right) d\beta \right\}$$

The Modular Duality of Form Summation Polynomial

Expanded Form Shown Below

$$E(\alpha + \beta)^n = f^0(\alpha^n) \cdot \int_0 \frac{\beta^0}{0!} d\beta + f^1(\alpha^n) \cdot \int_0^{\beta^0} d\beta + f^2(\alpha^n) \cdot \iint_0^{\beta^0} d\beta + f^3(\alpha^n) \cdot \iiint_0^{\beta^0} d\beta + \dots \rightarrow n$$

An expanded form of the polynomial to the (n^{th}) dimension is shown directly above where the successive *derivatives* of (α^n) are shown as $f^k(\alpha^n)$ and successive *integration* is shown as terms of the *successive integration polynomial* derived in the middle part of this section.

We could have just as easily substituted the *permutation* form $\left(\frac{n!}{(n-k)!} \alpha^{n-k} \right)$ of the

successive derivatives for $f^k(\alpha^n)$, however the shortened version was used, (lazy habits are hard to break).

In each term of the polynomial, the beta (β^0) *Productive Capacity* presence becomes an *expanding* successive *integration*, and the alpha (α^n) *Generative Power* presence becomes a *contracting* successive *derivative*. Recall the original definition of a binomial in the introductory text that *contraction* and *expansion* inferred (“*form* or *structure*”). We now have a *contracting* and *expanding* α, β oppositional polynomial using only *integral* and *differential* operators.

Because each term now contains only interactive successive (α^n) *derivatives* and $\left(\frac{\beta^0}{0!} \right)$

integration; it forms a *Modular Summation Equation*. By the word “*Modular*,” I mean that each

dimensional oppositional polynomial term is an independent, *stand alone*, “Module” of The *Duality of Form* summation polynomial, (which will be discussed more in *Section 5*).

Notice in both the expanded dimensional polynomial and the summation binomial above, each term has successive integration and differentiation (“*expansion* and *contraction*”), and the first term, which is referred to as a constant term, consistent in polynomial *terminology*, is actually the *0-derivative*, *0-integral* term ($k = 0$), and when ($k = 0$) no *integration* or *derivatives* are performed essentially making it a polynomial *constant* term. Notice also the *equivalency* in that for every successive *derivative* in a dimensional polynomial binomial module there is a corresponding equivalent successive *integration* in the module also.

In polynomials, the $f(e^\beta)$ *Productive Capacity* successive *integration* multiplicand, along with the successive *derivative*, is guiding nature’s dimensional evolutionary structure by forming a *Nature-ordered* complementary structure of the math. As the *derivative’s* (α^n) *exponent* is decreased, (*contracted*) by one integer unit, then similarly, the *integration’s* (β^0) *exponent* is increased (*expanded*) by one integer unit, consistent within a classic *binomial expansion*, and a *damping* equivalent transfer of *energy* in the process. The result in each term is the evolutionary integration’s passive element evolving (*expanding*) and the derivative’s volatile element decreasing, (*contracting*) to zero (0), producing a holistic dimensional evolutionary change in the α, β module/term. The modular summation equation can be used as the expansion of the form (*structure*) of an oppositional binomial. It can also be used in any operation involving *Base 2* oppositional dualities. The holistic operation defines the relationship ($\alpha \downarrow: \beta \uparrow$) and its’ opposite ($\beta \downarrow: \alpha \uparrow$).

Each polynomial term begins at a first interaction, *zero integration* and *zero derivatives* alike, and through the same amount of successive differentiation and integration in that exponential term; it ends with the new elements of the α, β oppositional change. This is very prominent in a binomial dimensional expansion, which we shall soon see, as oppositional ‘binomial *Lattice Datum* type dimensional space data’ and the *Hermetic Alchemical* $(\alpha + \beta)^n$ are transformed with a Dimensional Expansion Polynomial $E(\alpha + \beta)^n$; therefore its name, “The *Duality* of

Form polynomial." We'll discuss more interactions in *Section 5* where we will show each dimension's *presences* or *elements* and several more relationships.

Continued in Section 5

